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Publication date:
1990

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RESEARCH MEMORANDUM

DECISION-MAKING ON PENSION SCHEMES: EXPECTATION-FORMATION UNDER DEMOGRAPHIC CHANGE

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FEW 449

Decision-making on pension schemes:
expectation-formation under demographic change*

by

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Abstract - In this paper the formation of expectations during decision-making processes on pension schemes is the main focus. Politicians decide on the intergenerational transfer scheme and the young decide on savings. The politicians display Nash-behavior towards future generations of politicians and are the Stackelberg-leaders with the current young as followers. No generation of decision-makers is committed to previous decisions. If the exogenous parameters of the system are constant the benefit rate and the saving rate will become constant within a finite time interval provided a public pension system exists in the stationary-state. Under perfect foresight, a calculation of finite length suffices for the current decision-makers to generate the path of taxes and savings towards the stationary-state. The effects of demographic change can then be calculated as well.

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August, 1990

* We have benefited from comments received from participants of the ISPE-seminar on "The fiscal implications of an ageing population", held in Vaals, the Netherlands, May 30 - June 1, 1990. In particular, we thank Dieter Bös, Reiner Dinkel, Laurence Kotlikoff, Pierre Pestieau, Carol Propper and David Wildasin for useful comments on an earlier draft.

1. Introduction

In most developed countries, a substantial part of the elderly's income is provided for by means of public and private pension schemes. Generally, public pension plans are financed by a Pay-As-You-Go (abbreviated from now on as PAYG) system where current pension payments are financed by current tax payments. Fund raising as in a Capital Reserve (abbreviated from now on as CR) system, in principle, does not occur. On the other hand, private pension schemes are typically financed by a CR-system.[1] They are generally decided upon during wage negotiations. In this paper we will assume that these wage negotiations are decentralized and that no coordination between the different decision-making processes is possible.

The operation of PAYG-systems has drawn a great deal of attention recently due to the aging of the population which has become apparent since the 1970's. Because of this aging the long-run effects of PAYG-systems compared with CR-systems may be to reduce the lifetime utility of future generations. In view of this it is natural to ask in what sense public pension schemes and private pension schemes interact in the face of demographic change. This paper deals with the interaction between PAYG-financed old-age transfers and private pension schemes.

The interaction between private and public pension schemes has already been studied to a certain extent, for example in the paper by Boadway and Wildasin (1989). They used a median-voter framework in an overlapping-generations model, for the first time introduced in the literature by Samuelson (1958). In the Boadway-Widasin paper individuals are constrained in the sense that they cannot borrow against future public pension benefits. The probability of the constraint being binding, and thus keeping individuals off the optimal consumption path, obviously depends on the level of the tax rate. In particular, the higher the tax rate, the lower individual savings will be and the higher the probability that the individual will be constrained to borrow. If the borrowing constraint is binding, individuals prefer to consume more now instead of

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1) An exception to this case is France. Moreover, in most OECD countries the (private) pension schemes of public sector employees are run on a PAYG-basis.

when they are retired, and thus the benefit level (or the tax rate) preferred by the median voter will be negatively related to the benefit level of the public pension scheme (or tax rate) in existence up to the time a new decision has to be made. This feature ensures interior solutions of the tax rate. Another fundamental assumption that drives their dynamic results is that individuals believe that a tax rate, once established, will remain unchanged forever. Boadway and Wildasin find that a cycle of alternating over- and undershooting tax rates comes about which gradually damps down. This seems to fit the actual development of public pension schemes in many countries, where after a gradual expansion of PAYG-schemes recently measures have been taken to lower the level of intergenerational transfers. But, these measures have been inspired by the fall in population growth that cannot occur in the Boadway-Wildasin model (see e.g. Thompson, 1983). The cycle that comes about in their model is closely related to the way expectations are treated. Since individuals assume tax rates to be constant, they will base their consumption and saving plans on the tax rate at the time of planning. Therefore, if an individual is faced with a high tax rate at the start of life, he will start out with little savings. As a consequence, in the next period of his life he will want to have a relatively low tax rate to increase his second-period consumption and the other way around if the tax rate in the first period of his life is low. So, the cycle occurring in the Boadway-Wildasin model can be ascribed to the wrong expectations held by young generations.

Another interesting paper that deals with the interaction between public pensions and savings is the paper by Hansson and Stuart (1989). They use a two-overlapping-generations model to consider the evolution of public pension schemes. In their paper transfers are motivated by altruism. The assumption is made that decisions on public pension schemes are inherently constitutional, which in the two-generations model implies that a living generation can block legislation that would make it worse off. At the time of the introduction of the public pension scheme the living generations, who have perfect foresight, decide on the value of the transfers to the old and the savings by the young for the current time period and all future time periods. It is shown that a scheme will be chosen that cannot be amended by any future pair of generations and in which, furthermore, savings will go to zero, or, in other words, successive increases in the benefit level of public pension schemes will occur.

In actual fact public pension schemes are seldom part of the constitution. Every period the decision-makers are free to change the system to their own discretion and, indeed, public pension schemes are frequently changed. Obviously, if such changes are possible the Hansson-Stuart framework does not necessarily give a clue to what the future development of public pension schemes will be. Moreover, as stated above, many changes in public pension plans are inspired by demographic changes. But, just as in the Boadway-Wildasin paper, Hansson and Stuart assume a constant rate of population growth. Another important point is that in the Hansson-Stuart paper the decisive generations know by what mechanisms all other generations are driven in both their savings and transfer decisions. But, in reality, these two decisions are separated from one another. In principle, decision-making on intergenerational transfers takes place in political processes, while, as mentioned above, decisions on private pension schemes are generally determined during wage negotiations. Obviously, decision-makers in one field do not necessarily have all the information on the decision-making process in the other field. Indeed, one of the points of this paper is that for the development of public and private pension schemes in the face of demographic change it is of essential importance to know to what degree both sides of the decision-making process have information of the other side.

A paper that deals with the effects of demographic change is the paper by Boadway, Marchand and Pestieau (1990). (A similar framework is developed by Peters, 1990.) They consider a government that wants to maximize the undiscounted sum of utilities of successive generations. They show that under demographic changes and productivity fluctuations it will be optimal to smooth the development of lifetime incomes and utilities of the successive generations. This is accomplished by a system of social security benefits which is financed by a PAYG-system. If it is possible to borrow from the rest of the world the smoothing can be much larger, while if taxes and transfers lead to excess burdens the smoothing will be less. Boadway, Marchand and Pestieau consider the problem from a normative point of view. This implies that the governments succeeding the planner are assumed not to deviate from the optimal path, i.e. it is possible for the planning government to commit its successors (see also Boadway, Marchand and Pestieau, 1989, p. 12). As is already emphasized above in the context of the Hansson-Stuart paper, this assumption seems wholly unjustified in a positive framework.

From the above survey it can be concluded that the role of expectations is not yet satisfactorily modelled in the literature concerning public pension schemes. Therefore, this will be the main focus of this paper. We start from the assumption that every period a decision is made on the intergenerational transfer scheme by politicians and a decision is made by the private pension funds on the size of savings. The private pension funds are supposed to be effectively representing the young individuals' interests, which implies that from now on the concepts of private pension funds and generation of young individuals can be used interchangeably when decision-making is concerned. In taking these decisions the decision-makers are not committed in any way to measures taken by past generations. In this sense our paper differs from the constitutional framework adopted by Hansson and Stuart and the normative approach of Boadway, Marchand and Pestieau. The horizon of the decision-makers is assumed infinitely long. In contrast with the Boadway-Wildasin paper we assume that the decision-makers have perfect foresight. Because no generation of decision-makers is committed to previous decisions, every generation has to build an expectation of the outcome of the future decision-making process for itself. As, however, this outcome is in turn determined by the expectations held by the next generation of decision-makers, an in principle endless sequence of expectations has to be formed before a generation can take a decision of its own. But, in a situation in which the exogenous parameters of the system, such as demographics, productivity and real rate of interest, become constant after a certain time period, the benefit rate and the saving rate may become equal to constant values as well within a finite time interval. For the model presented in this paper it can be proved that this is actually true if a public pension scheme exists in the stationary-state. Under the assumption of perfect knowledge about the exogenous variables, the decision-makers will in that case also be able to foresee the emergence of this stationary-state. Given that knowledge, a calculation of finite length suffices to generate the path of taxes and savings towards the stationary-state. So, in our framework decision-makers are able to base their decisions on correct beliefs about the future development of the system, without having to impose ad hoc restrictions on the possible outcomes of the system.

The perfect foresight that is attributed to decision-makers in our model can be of two kinds. First there is what we will call ex-post perfect foresight: a generation can correctly calculate the development of benefit and saving rates and take a decision based upon the outcome of these calculations. Besides that, however, decision-makers can also have perfect foresight in an ex-ante sense. In that case, the decision-makers establish the relationship between the decisions they themselves have to make and decisions taken by other current or future decision-makers, by making use of the calculations that generate the ex-post perfect foresight outcomes. The knowledge about this relationship can then be used to manipulate the decisions by others. Ex-post perfect foresight corresponds to Nash-behavior, where decisions are made while taking the other decisions as given. Ex-ante perfect foresight, on the other hand, corresponds to Stackelberg-behavior, where decisions are made taking the decision-making processes of other agents into account.

In this paper two groups of decision-makers are distinguished, i.e. the politicians deciding on benefits for the old and the young deciding on savings. In principle, these two groups can display Nash- or Stackelberg-behavior towards each other. In other words, they can have ex-post or ex-ante expectations about the decisions of the other group of decision-makers. But expectations of future decisions also play a part in the current decision-making process. These can, in principle, also be of an ex-ante or an ex-post nature. Since we assume decentralized and uncoordinated decision-making by the young, the influence of each young decision-maker on the behavior of politicians is negligibly small. This rules out the possibility for the young to display Stackelberg-behavior towards future or current politicians. That leaves open four cases that may fit our model. To begin with, the current politicians can display Nash-behavior towards both future generations of politicians and the young. Secondly, they can display Nash-behavior towards future generations of politicians, while being the Stackelberg-leaders with the young as followers. Thirdly, they can be the Stackelberg-leaders towards future generations of decision-makers and behave Nash-like towards the young. Finally, the current politicians can be Stackelberg-leaders with both future generations of politicians and the young as followers. As will be discussed in section 6, the third and fourth case will lead to problems if the future generation of politicians can be assumed to be Stackelberg-leaders in turn. In the first case, if the politicians can calculate what the decisions made by young individuals will be, they also will be able to use these calculations to solve their own optimization problem. In other words, it is hard to imagine why in this case politicians can have ex-post expectations without necessarily also having ex-ante expectations. Therefore, most attention will be given in this paper to the second case.

The set-up of this paper is as follows. In section 2 the basic model will be introduced. In this model the young will be supposed to allocate consumption to different periods by maximizing a lifetime utility function. The politicians maximize a utility function in which the utility of the old and the young are combined. In section 3 the stationary-states will be derived. Section 4 describes the effects of a temporary demographic shock. In section 5 the economic interpretation of the model will be considered. Section 6 contains concluding comments.

2. The model

We start from the well-known two-overlapping-generations model where every individual lives for two periods. An individual born at time t will be endowed with one unit of a good at the start of life. A non-negative part of this endowment has to be transferred to the government (τ_t), part of it is used for savings (s_t, which also cannot be negative and which will earn no interest) and the remainder is used for consumption (c_t^y). So we get

$$c_t^y = 1 - s_t - \tau_t \tag{1}$$

When old, the individual consumes its savings and the transfer payment η_t to be received from the government:

$$c_t^r = s_{t-1} + \eta_t \tag{2}$$

The population born at time t is assumed to be composed of identical individuals and is of size N_t. The development of the population is dictated by the relation $N_t = n_t N_{t-1}$, where n_t is exogenously determined.[2)] The government has as its only activity providing transfers to the old which are financed by taxing the young. Assuming no administrative costs the budget restriction of this PAYG-system reads $\eta_t N_{t-1} = \tau_t N_t$ which implies

$$\eta_t = n_t \tau_t \tag{3}$$

We assume that an individual born at time t has a lifetime utility function that reads

$$\begin{aligned} U_t &= u(c_t^y) + u(c_{t+1}^r) \\ &= u(1 - s_t - \tau_t) + u(s_t + n_{t+1}\tau_{t+1}) \end{aligned} \tag{4}$$

where $u' > 0$, $u'' < 0$ and $u'(0) = \infty$ and the second equality follows after inserting eqs. (1) through (3). So, there is no time preference within a generation and no account is taken of the utility of other (past or future) generations. In such a framework old individuals will, of course, prefer a maximum transfer. Young individuals, on the other hand, will not consent voluntarily to contribute to a transfer system, provided they do not perceive a positive relation between the contributions they pay when they are young and the transfers they receive when they are old. As stated above, the government in this model takes care of the transfers

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2) Note that because one period equals a generation, the average age structure of a generation over one period is not influenced by the development of the birth rates. This implies that there will not be an autonomous cyclical reaction of the population growth to an initial exogenous change, as would be the case if shorter time periods were considered. On the other hand, there is an endogenous reaction to an autonomous change in the population growth rate one period after the change. This is due to the slow reaction of the population size to a change in the birth rates, as one generation only covers half of the total population. We will however abstract from this effect and thus assume the population growth rate to be fully exogenous.

from the young to the old. In taking its decisions the government is under pressure of both the old and the young to take account of these generations' interest. It will be assumed that the degree to which the government represents the interests of both groups is exogenous. In particular, politicians at time t act in accordance with the maximization of a weighed combination of the utility functions of the old and the young generation[3)]

$$
\begin{aligned}
D_t &= \lambda u(c^r_{t-1}) + (1-\lambda)\ n_t\{u(c^y_t) + u(c^r_t)\} \\
&= \lambda u(s_{t-1} + n_t\tau_t) + (1-\lambda)\ n_t\{u(1 - s_t - \tau_t) + u(s_t + n_{t+1}\tau_{t+1})\}
\end{aligned}
\tag{5}
$$

where the parameter λ indicates the effective weight of an old individual in the political process and the second equation is derived from the first by using eqs. (1) through (3). In this model, if the old are in the majority a maximum transfer need not necessarily come about as long as $\lambda < 1$.

Savings are determined by the young by maximizing eq. (4) subject to the constraint of non-negative savings. The first-order condition for this maximization problem is

$$
u'(1 - \tau_t - s_t) \geq u'(s_t + n_{t+1}\tau_{t+1}) \tag{6}
$$

where the inequality sign holds for zero savings. Note that the tax rates τ_t and τ_{t+1} have been taken as given. In this simple case eq. (6) boils down to

$$
s_t = \max\ \{0,\ \tfrac{1}{2}(1 - \tau_t - n_{t+1}\tau_{t+1})\} \tag{7}
$$

Obviously, savings depend on future tax rates. In particular, if a high future benefit is expected such that $\eta_{t+1} > 1 - \tau_t$, savings will be zero.

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3) See Cukierman (1990) and Verbon (1988) for a justification of this function. Apart from the interest function approach adopted here, the equation can just as well be interpreted as reflecting altruism of young individuals, see Verbon (1988).

Taking the saving rate s_t as given, transfers follow by maximizing (5) under the restriction that they cannot be negative. The first-order condition for this problem reads

$$\lambda u'(s_{t-1} + n_t\tau_t) \leq (1-\lambda)\ u'(1 - s_t - \tau_t) \tag{8}$$

where the inequality sign holds if $\tau_t = 0$. Notice that the size of the tax rate at t is no direct function of future decisions, but depends on current and past savings rates (s_t and s_{t-1}) and demographics (n_t) only. Moreover, if the political power of the old, measured by λ is smaller than the political power of the young (i.e. $\lambda < \frac{1}{2}$), the current consumption of the latter group will be higher than of the first group, while the reversed holds for $\lambda > \frac{1}{2}$.

Eqs. (7) and (8) determine the solutions for τ_t and s_t under Nash-behavior. In the context of our model, however, both (groups of) agents know exactly how the other party determines the instrument variable under their control (s_t for the young and τ_t for the politicians). But, as already has been discussed in the introduction, the institutional setting underlying our model prevents the young individuals from taking advantage of their knowledge, because the decision-making on the saving rate is decentralized and cannot be coordinated. The politicians, on the other hand, do not face such institutional restrictions. Therefore they can exploit their knowledge of the young individuals' decision-making process, that is their knowledge of eq. (7), to manipulate the young's decisions. That means that when the politicians determine the tax rate, they will explicitly take into account how their decision influences the size of the savings via eq. (7). In other words, the politicians are the Stackelberg-leaders in a game with the current generation of savers. Then we get instead of eq. (5)

if $\frac{1}{2}(1 - \tau_t - n_{t+1}\tau_{t+1}) \geq 0$ (see eq. (7)),

$$D_t = \lambda u(s_{t-1} + n_t\tau_t) + 2(1-\lambda)\ n_t u\{\tfrac{1}{2}(1 - \tau_t + n_{t+1}\tau_{t+1})\} \tag{9a}$$

if $\frac{1}{2}(1 - \tau_t - n_{t+1}\tau_{t+1}) < 0$,

$$D_t = \lambda u(s_{t-1} + n_t\tau_t) + (1-\lambda)\ n_t\{u(1 - \tau_t) + u(n_{t+1}\tau_{t+1})\} \tag{9b}$$

It can readily be verified that if eq. (9a) holds the first-order condition now reads instead of (8)

$$\lambda u'(s_{t-1} + n_t\tau_t) \le (1-\lambda)\ u'\{\tfrac{1}{2}(1 - \tau_t + n_{t+1}\tau_{t+1})\} \tag{10}$$

while if eq. (9b) holds condition (8) with $s_t = 0$ determines τ_t. Thus, if the politicians are the Stackelberg-leaders and the young have positive savings, the future of the public pension scheme is of importance to the political decision-making process. Changes in the expected future tax rate will then lead to a change in the current tax rate.

3. Stationary-state results

Let us suppose that in every period decisions are made according to the first-order conditions for τ_t and s_t. If the rate of population growth is a constant $n_t = n$, s_t and τ_t may become equal to constants $s_t = s$ and $\tau_t = \tau$, respectively (in the following, when the subscripts referring to time are omitted the stationary-state value is meant). In this section we establish what these stationary-states will look like in the model described in the previous section assuming that they actually exist. First, we will consider the Nash-case in which both young individuals and politicians take the other's decisions as given. Then we will work out the case where the current politicians are the Stackelberg-leaders towards the current young. Moreover, we will consider the conditions under which a stationary-state actually exists and whether it will be maintained if no exogenous changes occur.

In the Nash-case, the stationary-state is described by

$$\lambda u'(s + n\tau) \leq (1-\lambda)\, u'(1 - \tau - s) \tag{11}$$

for $\tau_t = \tau$ and

$$s = \frac{1}{2}(1 - \tau - n\tau) \geq 0 \tag{12}$$

for $s_t = s$. Obviously, s will be positive if and only if $\tau < 1/(1+n)$. Moreover, it appears from eq. (11) that whether a positive transfer will come about depends on the value of λ. It stands to reason that for high values of λ (large effective political power of the old) a positive value of τ is more likely to be generated than if λ is small. In particular the following proposition holds

> Proposition 1: In a stationary-state with Nash-behavior of politicians a positive transfer will exist if and only if $\lambda \geq \frac{1}{2}$. Moreover, if $\lambda < \frac{1}{2}$ in the stationary-state, then $s = \frac{1}{2}$. Finally, $s > 0$ and $\tau > 0$ is only possible when $\lambda = \frac{1}{2}$.

The proof of proposition 1 is straightforward. Suppose first $\lambda > 1-\lambda$ (or $\lambda > \frac{1}{2}$). Then, if $\tau = 0$, it will hold that $s = \frac{1}{2}$ and from condition (11) we derive $\lambda u'(\frac{1}{2}) \leq (1-\lambda)\, u'(\frac{1}{2})$ which implies a contradiction. So, $\tau > 0$. If $s > 0$ can be assumed, it follows from (11) and $\lambda > \frac{1}{2}$ that $1 - \tau - s < s + n\tau$ which, after inserting in condition (12), implies $\frac{1}{2}(1 - \tau + n\tau) < \frac{1}{2}(1 - \tau + n\tau)$, which is again a contradiction. So, $s = 0$. Suppose $\lambda < 1-\lambda$. Then if $\tau > 0$ and $s > 0$ is assumed, the same contradiction $\frac{1}{2}(1 - \tau + n\tau) < \frac{1}{2}(1 - \tau + n\tau)$ can be derived. So, $\tau > 0$ or $s > 0$. Suppose $s = 0$. Then $\tau \geq 1/(1+n)$. From eq. (11) it follows that $u'(n\tau) > u'(1 - \tau)$ which implies $\tau < 1/(1+n)$ which in turn is a contradiction. So $s > 0$ and $\tau = 0$, which implies (see condition (12)) $s = \frac{1}{2}$. Finally, suppose $\lambda = 1-\lambda$. Then it immediately follows that $\tau \geq 0$ and $s \geq 0$. QED.

In the case that the politicians are the Stackelberg-leaders, the first-order conditions in the stationary-state can easily be derived from eqs. (7) and (8). They turn out to be same as in the Nash-case, except for the first-order condition of the tax rates when $\frac{1}{2}(1 - \tau - n\tau) \geq 0$ holds, which reads, instead of eq. (11)

$$\lambda u'(s + n\tau) \le (1-\lambda)\ u'\{\tfrac{1}{2}(1 - \tau + n\tau)\} \tag{13}$$

It is straightforward to show the following proposition

Proposition 2: If the politicians know the decision-making process of the young, the stationary-state which is eventually reached will be of the same form as in proposition 1.

The proof of proposition 2 is completely analogous to the Nash-case, therefore it will be left to the reader to work it out. The results of proposition 1 and 2 are summed up in table 1.

Table 1. Stationary-state solutions in the Nash-case and Stackelberg-case

$\lambda < \frac{1}{2}$	$\lambda = \frac{1}{2}$	$\lambda > \frac{1}{2}$
$\tau = 0;\ s = \frac{1}{2}$	$\tau \ge 0;\ s \ge 0$	$\tau > 0;\ s = 0$

To summarize, in a stationary-state the existence of a transfer system depends solely on the political weights and not on the efficiency of the system.[4)] The reason for this is, of course, that the young generation takes the future tax rate as given and perceives no relation between current and future tax rates. In that case the transfer system is a pure loss for a young individual. If they have the larger weight, they are able to abolish the transfer system in the long run even if there exists one in the short run.

Two issues remain that have to be dealt with. The first issue is whether a stationary-state will be reached within a finite time interval if for one reason or another a situation off the stationary-state is occurring. In fact for the case in which transfers exist in the

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4) As is well-known, an intergenerational transfer system is intertemporally efficient if $n > 1$ under the conditions of the current model. Aaron was the first to make this condition more explicit, Aaron (1966).

stationary-state (i.e. $\lambda > \frac{1}{2}$), the equilibrium appears to be stable in the above sense. Notice that this stationary-state will be reached without having introduced a commitment technology as in Hansson and Stuart (1989). For the case in which transfers do not exist in the stationary-state (i.e. $\lambda < \frac{1}{2}$), stability is lacking. In particular, if the system gets off its stationary-state, a movement away from this state may be set in or a tendency in the direction of the stationary-state will arise. In the latter case the stationary-state will never actually be reached. As a result, for $\lambda < \frac{1}{2}$ determination problems arise for the current decision-makers in all cases. We formulate these results in proposition 3. Proofs and a more detailed interpretation are given in the next section.

> Proposition 3: If at some certain period the system is outside its stationary-state while the parameters are constant, then the development of the tax rates will be such that the stationary-state will be re-established within a finite time interval if $\lambda > \frac{1}{2}$. If $\lambda < \frac{1}{2}$ a return to the stationary-state is impossible within the context of our model.

The second issue is whether if the system happens to be in a stationary-state, this state will be maintained if no further exogenous shocks occur. We can state the following proposition

> Proposition 4: The stationary-states which have been described in proposition 1 and 2 are Nash equilibria between succeeding generations.

Proposition 4 can easily be checked. Take first the case $\lambda > \frac{1}{2}$, so that $s = 0$ in the stationary-state. Given $\tau_{t+1} = \tau$ the question is whether at time t the political process would generate a tax rate such that $s_t > 0$ would result. If not then, obviously, $\tau_t = \tau$ and a stationary-state would not be disturbed at time t. Assume $s_t > 0$. Maximizing decision-making function (9a) produces $\lambda u'(n\tau_t) = (1-\lambda)\ u'\{\frac{1}{2}(1 - \tau_t + n\tau)\} > (1-\lambda)\ u'(n\tau_t)$ so that $\frac{1}{2}(1 - \tau_t - n\tau) + n(\tau - \tau_t) < 0$, or $s_t < n(\tau_t - \tau)$. Obviously, $\tau_t \le \tau$ so that s_t cannot become positive and $\tau_t = \tau$ as a result. Consider next the case $\lambda < \frac{1}{2}$ so that $\tau = 0$ in the stationary-state. As $\tau_{t+1} = 0$,

$s_t > 0$ unless $\tau_t = 1$. If $\tau_t = 1$ (and $s_t = 0$) the first-order condition would read $\lambda u'(n) = (1-\lambda)\ u'(0)$ which obviously cannot hold. So $s_t > 0$. If $\tau_t > 0$ the first-order condition (10) would give $\lambda u'(\frac{1}{2} + n\tau_t) = (1-\lambda)\ u'\{\frac{1}{2}(1 - \tau_t)\} < (1-\lambda)\ u'(\frac{1}{2} + n\tau_t)$, which leads to the contradiction $\tau_t < 0$. So in both cases $\lambda > \frac{1}{2}$ and $\lambda < \frac{1}{2}$ no generation of politicians would have an incentive to leave the stationary-state once it is reached. As the young in their saving decisions follow the politicians, they will not deviate from the stationary-state either. QED.

Note, finally, that if $\lambda < \frac{1}{2}$ according to proposition 3 a stationary-state will never be reached if an off-stationary-state situation is occurring. On the other hand, proposition 4 says that if the system happens to be in a stationary-state, there is no endogenous way out of it. In the next section it will be made clear that exogenous changes in the rate of population growth cannot disturb the no-transfer stationary-state either.

4. Comparative-statics results: effects of a temporary demographic change

In reality, of course, no stationary-state can be observed. Actually, in the post-war period large changes in private and public pension systems occurred in the western world. Apart from other factors, changes in the rate of population growth is one of the driving forces behind these changes. More recently projected demographic changes have led politicians to enact measures which implied a cut in benefit rights of the public pension schemes.

In this section the effects of demographic changes in our model will be considered starting from the assumption that the system is in a stationary-state. We will only work out the case where the current politicians are the Stackelberg-leaders towards the current young. If the model is stable, demographic changes can only lead to temporary changes (if any) in transfer and saving rates as the stationary-state does not depend on the actual rate of population growth, but only on the political power weight λ. However, according to proposition 3 stability can only be guaranteed if $\lambda > \frac{1}{2}$, while if $\lambda < \frac{1}{2}$ an off-stationary-state situation can never be restored. So, if our model has any explanatory value, the observed changes in pension systems should be interpreted in terms of the former case where $\lambda > \frac{1}{2}$. Because of stability, the effects of temporary shocks will then dampen out if no new shocks in demographics will occur.

Important to the argument of this section is the assumption that the relevant decision-maker has perfect foresight with respect to the decisions that are to be taken in the future, given the demographic shock. In particular, if at time t it has become known for the first time that a demographic change will occur at time t+j, the rational decision-maker at time t can calculate whether this change will lead to a deviation from the stationary-state at times t+j or t+j-1. If not, the demographic change will not have any effect anyhow. This will appear to be the case in what follows below if $\lambda < \frac{1}{2}$. So, only if $\lambda > \frac{1}{2}$ the stationary-state can be disturbed due to a demographic shock. This perturbation can have forward and backward effects. But at some future time period the stationary-state will establish itself again because of stability in the case where $\lambda > \frac{1}{2}$. Then a finite calculation is sufficient for the current generation to calculate the optimal decision on its own tax or saving rate. The current generation has to write down the comparative-statics equations to be derived from the first-order condition in order to calculate the future effects. These future effects and the consequent current effects will now be calculated for the model under consideration.

In the Stackelberg-case the politicians determine the development of the transfer system, while the young follow with their savings. So, we only need to keep track of the tax rates. Four cases can be distinguished, i.e. a positive demographic shock for $\lambda < \frac{1}{2}$ and $\lambda > \frac{1}{2}$, respectively, and a negative demographic shock for $\lambda < \frac{1}{2}$ and $\lambda > \frac{1}{2}$. First take the case in which the young have the larger political weight in the decision-making process, i.e. $\lambda < \frac{1}{2}$. A change in the rate of population growth in period t+j is expressed in the parameter $dn = dn_{t+j} = n_{t+j} - n \neq 0$. The first-order effect of the change in the rate of population growth on the savings at time t+j-1 equals, according to eq. (7), $\partial s_{t+j-1} = -\frac{1}{2}(n_{t+j}\partial\tau_{t+j} - \tau_{t+j}dn) = -\frac{1}{2}n_{t+j}\partial\tau_{t+j}$. If the first-order effect on the tax rate would be such that $\tau_{t+j} > \tau = 0$, then it follows from condition (8) that

$$\lambda\, u''(s_{t+j-1} + n_{t+j}\tau_{t+j})\, n_{t+j}\partial\tau_{t+j} =$$

$$-(1-\lambda)\, u''\{\tfrac{1}{2}(1 - \tau_{t+j} + n_{t+j+1}\tau_{t+j+1})\}\partial\tau_{t+j} \qquad (14)$$

which can only imply that $\partial\tau_{t+j} = 0$. So, the first-order effect of a positive or negative demographic change on the tax rate and the saving rate cannot differ from zero. This result is re-stated in the next proposition

<u>Proposition 5</u>: When in the Stackelberg-model with $\lambda < \frac{1}{2}$ the stationary-state with zero transfers exists, there is no way out of it by means of temporary shocks.

Next take the case $\lambda > \frac{1}{2}$, so that $s = 0$ and $\tau \geq 1/(1+n)$ in the stationary-state. It can be proved that outside the stationary-state the tax rates will be positive for all time periods.[5)] Moreover, as long as $s_{t+h} = 0$ for all $h \geq 0$ after the change dn has become known, the stationary-state tax rates will not change for $h \neq j-1$, j. Notice further that the expected changes in tax rates will be driven by their changes at time t+j and t+j-1: the effect of the demographic shock will only be felt directly at times t+j and t+j-1, while in the other periods the effects of the shock are felt indirectly through the changes in the tax rates of other time periods. It follows that the comparative-statics analysis has to start at time periods t+j and t+j-1. The argument proceeds along a number of steps. In every step a sequence of saving rates is assumed positive. As in the stationary-state savings are zero, it will not be disturbed at times other than t+j or t+j-1 if current and previous savings remain equal to zero. So

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5) Suppose at time period t the system is outside the stationary-state. If $s_t = 0$ and assuming $\tau_t = 0$, we get from condition (8) $\lambda u'(s_{t-1}) \leq (1-\lambda)\, u'(1) < \lambda u'(1)$, where the last inequality follows from $\lambda > \frac{1}{2}$. This produces the contradiction $s_{t-1} > 1$. On the other hand, if $s_t > 0$ condition (10) holds and assuming $\tau_t = 0$ we get $\lambda u'(s_{t-1}) \leq (1-\lambda)\, u'(\frac{1}{2} + \frac{1}{2} n_{t+1}\tau_{t+1}) < u'(\frac{1}{2} + \frac{1}{2} n_{t+1}\tau_{t+1})$ where $\lambda > \frac{1}{2}$ has been used again. It follows that $s_{t-1} > \frac{1}{2} + \frac{1}{2} n_{t+1}\tau_{t+1} \geq \frac{1}{2}$, which is a contradiction as $s_t \leq \frac{1}{2}$ must always hold, see eq. (7). QED.

all we need to do is to consider how a demographic shock at time t+j can initially change the tax rates at times t+j and/or t+j-1 and whether these changes can drive the saving rates to positive values. Regarding the interpretation of the ensuing analysis it should be stressed that the calculations are performed at time t. At that time, of course, the future is still open, and all deviations from the stationary-state have to be determined as yet. However, with perfect foresight the current generation of politicians knows the effects future and past changes of tax rates have on the tax rates holding at any time period. This knowledge is described by the comparative-statics equations.

Notice first that $s_{t+j} = s_{t+j-1} = 0$ before the demographic change. Assume these savings remain zero after the change as well. The first-order conditions for τ_{t+j} and τ_{t+j-1} then are of the form (8) with savings equal to zero. The direct effect of dn can be calculated as follows

$$(n_{t+j} + V_{t+j})d\tau_{t+j} = - \tau_{t+j}dn \tag{15}$$

where $V_{t+j} = \frac{1-\lambda}{\lambda} \frac{u''(c^y_{t+j})}{u''(c^r_{t+j-1})} > 0$, while $d\tau_{t+j-1} = 0$. So the initial effect of a demographic change will then be negative, i.e. $d\tau_{t+j}/dn < 0$. It follows immediately that if $dn < 0$, s_{t+j} cannot become positive so that, as a result, $d\tau_{t+h} = 0$ for $h > j$. This result holds through if $ds_{t+h} > 0$ for $h \le j-1$ as will become clear below.

Given the initial effect (15) we can now check whether $s_{t+j-1} > 0$ is possible. As a necessary condition for $s_{t+j-1} > 0$ it follows from (7) that $- d(n_{t+j}\tau_{t+j}) = - \tau_{t+j}\, dn - n_{t+j}d\tau_{t+j} < 0$ has to hold. Using (15) we find that the latter inequality always holds if $dn < 0$. So, assume next $s_{t+j-1} > 0$ and all other saving rates equal to zero. The first-order conditions for τ_{t+j-1} and τ_{t+j}, respectively, then read

$$\lambda u'(n\tau_{t+j-1}) - (1-\lambda)\, u'\{\tfrac{1}{2}(1 - \tau_{t+j-1} + n_{t+j}\tau_{t+j})\} = 0 \tag{16}$$

and

$$\lambda u'\{\tfrac{1}{2}(1 - \tau_{t+j-1} + n_{t+j}\tau_{t+j})\} - (1-\lambda)\, u'(1 - \tau_{t+j}) = 0 \tag{17}$$

The effects of the change in demography can then be calculated from (16) and (17)

$$(n + \frac{1}{2} V_{t+j-1})d\tau_{t+j-1} = \frac{1}{2} V_{t+j-1} \tau_{t+j}dn + \frac{1}{2} n_{t+j}V_{t+j-1}d\tau_{t+j} \quad (18)$$

$$(\frac{1}{2} n_{t+j} + V_{t+j})d\tau_{t+j} = - \frac{1}{2} \tau_{t+j}dn + \frac{1}{2} d\tau_{t+j-1} \quad (19)$$

From eqs. (18) and (19) it can be derived that $d\tau_{t+j}/dn < 0$ and $d\tau_{t+j-1}/dn > 0$. From the latter inequality it follows that if $dn > 0$ then s_{t+j-2} cannot increase so that, as a result, $d\tau_{t+h} = 0$ for $h < j-1$. So, going backward in time we can restrict ourselves to the case of a negative demographic shock. Extending the comparative-statics analysis underlying eqs. (18) and (19) it is easy to prove the following

> Proposition 6: If in the case where $\lambda > \frac{1}{2}$ a drop in the rate of population growth occurs at time t+j, then $d\tau_{t+j} > 0$. Furthermore $d\tau_{t+h} = 0$ for $h > j$. Moreover, if $s_{t+h} > 0$ for $h = k,\dots,j-1$ and $k \geq 0$, then $d\tau_{t+h}/dn > 0$.

The fact that a demographic decline has no effects after the date of the shock follows from the fact that at the date of the shock the savings do not become positive. As noted above, in this model changes in the tax rates can only occur (apart from demographic change) if the saving rate of the previous and/or next young generation changes. So, for the decision-making process after time t+j nothing has changed and the stationary-state will immediately be re-established. In table 2 the effect of demographic change is summarized for this case.

Table 2. The effects of a temporary negative demographic shock in the Stackelberg-case with $\lambda > \frac{1}{2}$

$h < j$	$h = j$	$h > j$
$\tau_{t+h} \leq \tau$; $s_{t+h} \geq s$	$\tau_{t+h} > \tau$; $s_{t+h} = s$	$\tau_{t+h} = \tau$; $s_{t+h} = s$

Let us next turn to the case of a positive demographic shock

> <u>Proposition 7</u>: If in the case where $\lambda > \frac{1}{2}$ a rise in the rate of population growth occurs at time t+j, then $d\tau_{t+h} = 0$ for $h < j-1$, $d\tau_{t+j-1} \gtrless 0$, $d\tau_{t+j} < 0$ and a finite number $K \geq j$ exists such that $s_{t+K-1} \geq s_{t+K} = 0$ and $d\tau_{t+h} < 0$ for $h = j+1,\dots,K$ and $d\tau_{t+h} = 0$ for $h > K$.

The first part of the proposition follows from the fact noted above that if $dn > 0$ then $s_{t+h} = 0$ for $h < j-1$. We also have that s_{t+j} can be positive if $dn > 0$. If $s_{t+j} > 0$ and all other saving rates are zero, then the effects of demographic change on τ_{t+j} is given by

$$(n_{t+j} + \frac{1}{2} V_{t+j})d\tau_{t+j} = - \tau_{t+j}dn + \frac{1}{2} nV_{t+j}d\tau_{t+j+1} \tag{20}$$

However, as s_{t+j+1} is zero, $d\tau_{t+j+1}$ is zero as well. It immediately follows from (20) that $d\tau_{t+j}/dn < 0$. However, we can check by evaluating $d(n_{t+j}\tau_{t+j})$ that $s_{t+j-1} > 0$ cannot be excluded. Applying the same comparative-statics analysis with $s_{t+j-1} > 0$ we find $d\tau_{t+j}/dn > 0$ as a possibility. However, in the latter case we find from (18) $d\tau_{t+j-1} > 0$ so that we have the contradiction $s_{t+j-1} = 0$. In other words, whatever the value of $s_{t+j-1} \geq 0$ the inequality $d\tau_{t+j}/dn < 0$ should hold. The same result holds through if future saving rates can be assumed positive. Future positive savings at times t+h ($h > j$) can be engendered by positive savings at times t+j. The fact that $s_{t+j} > 0$ implies $d\tau_{t+j+1} < 0$ and leads to the possibility that $s_{t+j+1} > 0$. So a chain of positive savings can arise due to the positive demographic change and the corresponding tax rates are below their stationary-state values. The proposition states that the time interval during which $s_{t+h} > 0$ and $d\tau_{t+h} < 0$ ($h > j$) occurs is finite. This final part of the proposition can be proved by a comparative-statics analysis as above. Suppose that $s_{t+j},\dots,s_{t+j+k-1}$ are positive for $k > 1$, and $s_{t+j+k} = 0$. This implies that the first-order conditions with respect to the tax rate read

$$\lambda u'\{\tfrac{1}{2}(1 - \tau_{t+h-1} + n\tau_{t+h})\} - (1-\lambda)\ u'\{\tfrac{1}{2}(1 - \tau_{t+h} + n\tau_{t+h+1})\} = 0 \qquad h=j+1,\ldots,j+k-1 \tag{21}$$

and

$$\lambda u'\{\tfrac{1}{2}(1 - \tau_{t+H-1} + n\tau_{t+H})\} - (1-\lambda)\ u'(1 - \tau_{t+H}) = 0 \tag{22}$$

where $H = j+k$. Eqs. (21) follow from eq. (9b) after inserting the savings of the previous generation in the first-order condition. In eq. (22) account is taken of the fact that $s_{t+j+k} = 0$.

For the tax rates as from time t+j+1 the effects of the demographic change purely follow from future and/or previous changes in the tax rates. These effects can be calculated by totally differentiating eqs. (21) and (22) which gives:

$$(n + V_{t+h})d\tau_{t+h} = d\tau_{t+h-1} + nV_{t+h}\ d\tau_{t+h+1} \qquad h=j+1,\ldots,j+k-1 \tag{23}$$

and

$$(\tfrac{1}{2}\ n + V_{t+H})d\tau_{t+H} = \tfrac{1}{2}\ d\tau_{t+H-1} \qquad H = j+k \tag{24}$$

The effects on the tax rates at times t+h (h > j) can be calculated as a function of the change in the tax rate at time t+j by recursive substitution. For instance, for time t+j+k we get

$$d\tau_{t+j+k} = \tfrac{1}{2}\ d\tau_{t+j}/A_k \tag{25}$$

where

$$A_k = \tfrac{1}{2}\ n^k + \sum_{h=1}^{k} n^{k-h} \prod_{i=1}^{h+1} V_{t+j+i} \tag{26}$$

while for $d\tau_{t+h}$ (h=j+1,...,k-1) analogous expressions can be derived. From these expressions it follows that the direction of the change in τ_{t+h} (h > j) equals the direction of the change in τ_{t+j}, so that $d\tau_{t+h} < 0$. We finally have to prove that the saving rate will reach the value zero within a finite time interval. This follows immediately if $n > 1$ as then the expression A_k goes to infinity for $k \to \infty$. The negative effects of the demographic change on future tax rates will become infinitely small in the course of time which implies that for some time t+K savings are no longer positive, i.e. $s_{t+K} = \frac{1}{2} - \frac{1}{2}\tau_{t+K} - \frac{1}{2}n\tau_{t+K+1} \leq 0$. From that period onwards the stationary-state will be re-established. If $n < 1$ the expression A_k could approach zero for increasing k. This implies that the actual tax rates must be decreasing. So savings will remain positive and condition (21) holds from which we derive the inequality

$$\tau_{t+h} - \tau_{t+h-1} > n(\tau_{t+h+1} - \tau_{t+h}) \tag{27}$$

Inequality (27) states that if the tax rates decrease, the direction of the future change cannot be reversed. Then the tax rate will become zero which is a contradiction as the tax rate will always be positive outside the stationary-state (see footnote 5). So, A_k cannot converge to zero. It cannot converge to a finite constant either, because in that case the tax rate converges to a constant while savings are positive and the factor V_{t+j+i} in eq. (26) will approach the value $\frac{1-\lambda}{\lambda} < 1$ so that, again, A_k would approach zero. In other words, in all cases the tax rates have to increase and savings will be zero within a finite time interval. QED.

In table 3 the effect of demographic change is summarized for this case.

Table 3. The effects of a temporary positive demographic shock in the Stackelberg-case with $\lambda > \frac{1}{2}$

$h < j-1$	$h = j-1$	$h = j$	$h > j$
$\tau_{t+h} = \tau$; $s_{t+h} = s$	$\tau_{t+h} \gtrless \tau$; $s_{t+h} \geq s$	$\tau_{t+h} < \tau$; $s_{t+h} \geq s$	$\tau_{t+h} \leq \tau$; $s_{t+h} \geq s$

Regarding the path of the tax rates in the both cases of a negative and positive demographic shock, respectively, we can establish

Proposition 8: For negative and positive demographic shocks the tax rates will move continuously in the same direction. In particular, $\tau_{t+h+1} < \tau_{t+h}$ for $h < j$ in case of a negative demographic shock and $\tau_{t+h+1} > \tau_{t+h}$ for $h > j$ in case of a positive demographic shock.

The proof of proposition 8 uses inequality (27) for both negative and positive demographic shocks. Take first the case $dn_{t+j} < 0$. Suppose at time t+k savings become positive, i.e. $s_{t+k-1} = 0$ and $s_{t+k} > 0$ for $0 \leq k < j$. Then from the first-order condition at time t+k (see eq. (16) with $k = j$) we derive $n\tau_{t+k} > \frac{1}{2}(1 - \tau_{t+k} + n\tau_{t+k+1})$ or $n(\tau_{t+k} - \tau_{t+k+1}) > \frac{1}{2}(1 - \tau_{t+k} - n\tau_{t+k+1}) > 0$. So, $\tau_{t+k+1} - \tau_{t+k} < 0$. But then, according to (27) $\tau_{t+h+1} - \tau_{t+h} < 0$ as well for $k < h < j$. For the case of a positive demographic shock, assume savings become zero at time t+k, i.e. $s_{t+k-1} > 0 = s_{t+k}$, for $k > j$. From the first-order condition at time t+k (see eq. (22) with k = H) we derive $\frac{1}{2}(1 - \tau_{t+k-1} + n\tau_{t+k}) > 1 - \tau_{t+k}$, or $\frac{1}{2}(1 - \tau_{t+k-1}) > 1 - \tau_{t+k} - \frac{1}{2} n\tau_{t+k}$. Moreover, because $s_{t+k-1} > 0$, we also have $\frac{1}{2}(1 - \tau_{t+k-1}) > \frac{1}{2} n\tau_{t+k}$. Combining the last two inequalities we find that $\tau_{t+k} - \tau_{t+k-1} > 0$. But then, according to (27), it should also hold that $\tau_{t+h} - \tau_{t+h-1} > 0$ for $j < h < k$. QED.

As an illustration of propositions 6, 7 and 8 figures 1 and 2 give typical paths of tax and saving rates under demographic change.

insert figures 1 and 2 here

As a byproduct of the proof of proposition 7 we can also give a proof of proposition 3. In particular eqs. (25) and (26) give the evolution of the tax rate τ_{t+k} for any perturbation $d\tau_t \neq 0$. So, if the system gets off its stationary-state and $\lambda > \frac{1}{2}$, then the stationary-state will be restored within a finite time interval. Actually this is a basic result of our model. It ensures that the effects of (demographic) shocks will be of a finite length so that decision makers gifted with perfect foresight can actually calculate what course of action they have to take themselves. This is, however, not the case if $\lambda < \frac{1}{2}$. From eqs. (25) and (26) we see that $d\tau_{t+k} \to 0$ if $n > 1$, never actually reaching the stationary-state. If $n < 1$, the development may even be away from its stationary-state $\tau = 0$, $s = \frac{1}{2}$. The state $s = 0$, $\tau > 0$ will not be an equilibrium as savings cannot remain zero for more than one period. Thereafter the tax rate will decrease and may keep on decreasing, but also in that case it cannot return to the stationary-state due to eq. (25). So, when $\lambda < \frac{1}{2}$ the stationary-state will never be restored within a finite time interval. Then the whole expectation-formation mechanism breaks down as an infinite number of calculations is in that case needed to calculate the effects of demographic shocks. In other words, the current decision-makers are not able to choose an optimal tax rate as an unambiguous path of future tax and saving rates is not available. Note, however, that for the case where $\lambda < \frac{1}{2}$ demographic shocks do not have any effect anyhow. So, we have the paradoxical case in which a stationary-state cannot be restored if it is broken, but if the system is in the stationary-state, it cannot be broken by exogenous shocks. Finally, it should be remarked that the lack of stability or the lack of convergence within a finite interval for this case where $\lambda < \frac{1}{2}$ might be due to peculiarities of our model. However, it might also be a general characteristic of overlapping-generations models where generations are not linked to each other by public or private transfers.

5. The economic interpretation of the model

The results that have been derived in the sections 3 and 4 are very general in the sense that they do not depend on a specific utility function and that the modelling of the decision-making by the government can be interpreted in different ways (see footnote 3). Another feature of the model is the assumption of perfect foresight. As already has been pointed out in the introduction, one of the great advantages of assuming perfect foresight is that the expectations of agents can be modelled without making use of ad hoc assumptions. Moreover, the way expectations are treated here makes it possible to easily calculate the stationary-states and dynamic paths the system will follow in different circumstances provided disturbances of the stationary-state will dampen out within a finite time interval which, as mentioned, in section 4, will only be true if transfers exist in the stationary-state. Admittedly, the model could improve in terms of plausibility if uncertainty would be introduced, especially since the model essentially describes the situation on the long run (one period equals a generation, see footnote 2). Although this might be an interesting extension, this is not done here as it would complicate matters considerably without adding anything essential to the argument of the previous sections. Furthermore, the problem of pensions is deliberately modelled in a highly stylized way to get a grip on the basic features. To give but one example, no excess burdens of taxation have been assumed. This is the major reason why in this model corner solutions are arrived at in the stationary-state. Interior solutions of tax rates and saving rates could occur simultaneously if the young might be able to decrease their endowment if confronted with an increase in the tax rate. Note that such excess burdens would prevent corner solutions from arising as well in median-voter models (see e.g. Meltzer and Richard, 1981).

With respect to the institutional setting underlying the model it should be pointed out that in assuming total discretionary power of politicians over the current tax rate only the model is implicitly limited to the case of a pure tax-transfer system as opposed to an insurance system. However, as demonstrated in Thompson (1983, see also Verbon, 1988, Ch. 6) the tax-transfer vision has become the dominant one in the actual operation of public pension schemes. More important is the way the supposed institutional framework influences the possibility of exploiting the perfect knowledge of the system by means of Stackelberg-behavior. By assuming decentralized and uncoordinated decision-making by young individuals it is excluded that these are the Stackelberg-leaders towards the politicians.[6] Another point is that it has been assumed until now that both young individuals and politicians cannot be the Stackelberg-leaders towards future generations of decision-makers, although this does not seem to be impossible at first sight. In fact this case has been worked out by Veall (1986). In Veall's paper the current generation of young individuals determines the savings and the tax rate in a game where they are the Stackelberg-leaders towards the next generation of young

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6) Alternatively, in a number of countries, notably the Netherlands, decision-making on private pension funds is centralized, which opens up the possiblity for the young to actually exploit their knowledge about the political decision-making process. See Verbon and Verhoeven (1990) for an extensive treatment of this case.

individuals, which is essentially the same as the current generation. In particular, the current young calculate the reaction of their successors under the assumption that the latter will decide while taking all future decision-making as given. This, however, seems inconsistent with the assumptions of perfect foresight and rational behavior of the current young, that underlie both this and Veall's paper. For, if the current young know the next generation's decision-making they will also know that the latter will be the Stackelberg-leaders towards the then next generation, which generation would in turn behave likewise. This implies that if the current young want to avoid time inconsistency, they have to be able to forecast the reaction of the next generation on current decisions. This reaction has to be known before the young take decisions themselves. However, this leads to an infinite loop as current decisions can only be taken if future reactions are known, while these future reactions in turn should be of the same form as the current decisions. We can conclude that Veall's approach is unsatisfactory and that the assumption of Stackelberg-behavior of current towards future generations in a framework of perfect foresight and rational behavior is not a consistent one.

In the light of the above remarks, we need to qualify our perfect foresight hypothesis. In the above argument it was implicitly assumed that perfect foresight referred to both the future outcomes of the system and the way these decisions will be arrived at, or, equivalently, that decision-makers had perfect foresight as regards to both ex-post and ex-ante expectations. So, apart from the actual path of future saving rates and tax rates (ex-post), decision-makers also have the information to calculate the path in alternative circumstances (ex-ante). From the above consistency argument it follows that we have to drop the assumption that politicians have the latter kind of information about future politicians, although they can have that information with respect to the current or future savers. They have no alternative but to take the future decisions of politicians as given. Conceptually, we could think of this as the introduction of a kind of Central Forecasting Bureau that is able to generate forecasts of tax rates and savings and transmits only the end results to the decision-makers.

We can conclude that the assumption of Nash-behavior towards future generations seems satisfactory, provided that there exists a certain institutional framework that guarantees that the decision-makers have the relevant information about future outcomes without giving them the opportunity to exploit this knowledge totally. Finally, as has already been mentioned in section 2, the existence of such a Nash-equilibrium can be proved.

The last point to be raised in this section concerns the exogeneity that is assumed with respect to the demographic and economic development and the institutional framework. Dropping this assumption indeed implies an interesting field of research. The omission of these aspects in this paper can however be warranted by the fact that we have limited ourselves here to a partial analysis aimed at the development of the pension system as such.

6. Conclusions

In this paper the evolution of public and private pension schemes represented by the tax rate of the public pension scheme and the saving rate of the young, respectively, has been studied. We assumed that decisions on these two schemes are taken by a political process that fixes the benefit and tax rates for public pensions, while the young determine independently from each other their saving rates. For the evolution of both schemes the expectations generations have of future decisions in a changing environment is of primary importance. One of the main interests for setting up this study is in trying to model these expectations in a consistent way, which is as far as we are able to see one of the main problems in the literature on public pension schemes. We model expectations in the following way. A distinction is made between ex-ante and ex-post expectations. As explained before, under ex-ante expectations the future can be manipulated by current decision-makers while under ex-post expectations the complete future is known but has to be taken as given. In our model both politicians and young individuals can have ex-ante information about the other party's contemporaneous decisions. But young individuals cannot make use of this information because of the decentralized character of their decision-making. Decision-makers can only have ex-post expectations with respect to future developments of the system. This set-up can only be useful, however, if these ex-ante and ex-post expectations can be actually calculated at the time of decision-making, i.e. if the system described in our model will return to a stationary-state after a disturbance of the equilibrium by, e.g. a demographic shock. This appears to be the case if transfers exist in the stationary-state. The path towards this stationary-state can then be calculated.

It appeared that intertemporal efficiency is of no importance in the stationary-state. Only the political power of the old and young generations, which are alive at the time the decisions are made, count in this respect. This is not too amazing, of course, because in the political process as we described it, public pension benefits only serve as a means to redistribute income from the young to the old.

Given the treatment of expectations described above we could consider the effects of possible future demographic changes in a stationary-state with a positive tax rate. As regards to a future demographic change, the effects of such a change on tax rates or saving rates, as mentioned above, appear to have a finite length after a change. Moreover, the paths are of a 'smooth' nature in the sense that before or after the change, -in anticipation of the effects calculated by the 'Central Forecasting Bureau'- the relevant instrument increases or decreases constantly until the shock occurs respectively the stationary-state is reached. So, no cycles occur in which decision-makers are continuously correcting their errors in expectations, but, on the contrary, decision-makers correctly anticipate the incorporation of the change in the tax or saving rate which is implemented by the next generation. The deviations from the stationary-state predominantly occur either before or after the demographic shock.

As a consequence of the above results, we can interpret changes in private pension schemes as temporary deviations from a stationary-state in which, in the context of our stylized model, no private pension scheme exists anyhow. Adaptation of private pension schemes is engendered by demographic (or other exogenous) changes. As negative demographic shocks disturb the stationary-state before the shock actually occurs and positive shocks exert their influence forwards, this interpretation suggests in the context of our model that private pensions exist when positive demographic shocks have been experienced in the past and/or negative shocks are foreseen in the future. The same interpretation, however, cannot hold for public pension schemes, as a stationary-state without public pensions cannot be broken by temporary exogenous shocks. As a consequence, for the existence of public pension plans a high level of political power of the old is needed (or, alternatively, a high level of altruism of young individuals, see footnote 3), since then there will be a stationary-state with positive transfers. This result corresponds to Browning (1975). The development of a mixed public and private pension system, that can be observed in actual fact, should then be explained by the occurrence of (positive demographic) shocks in the past and (negative demographic) shocks in the anticipated future.

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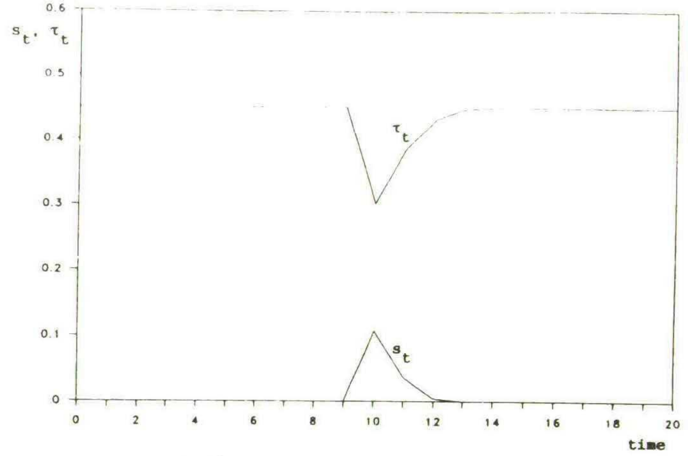
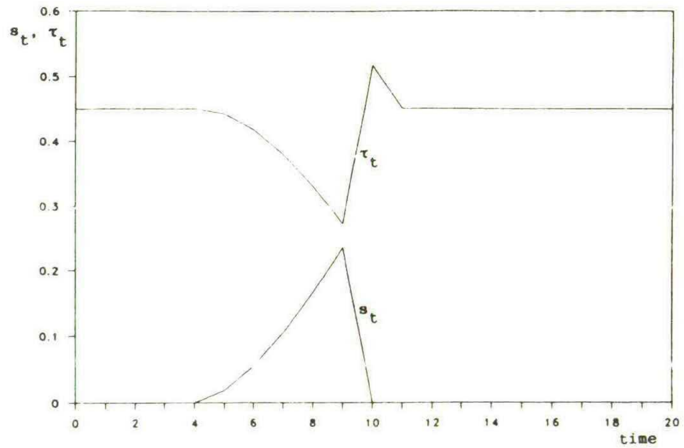
Figure 1. The effects of a negative population growth shock in the Stackelberg-case when $\lambda > \frac{1}{2}$



Note: $u(c) = -\frac{1}{8}c^{-8}$, $\lambda = 0.55$, $n_t = 1.25$ for all periods, except $n_{10} = 0.5$

Figure 2. The effects of a positive population growth shock in the Stackelberg-case when $\lambda > \frac{1}{2}$



Note: $u(c) = -\frac{1}{8}c^{-8}$, $\lambda = 0.55$, $n_t = 1.25$ for all periods, except $n_{10} = 2.0$

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